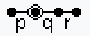
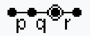
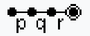
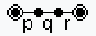
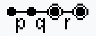
The *Coxeter-Dynkin diagram* shows the four mirrors of the Wythoffian kaleidoscope as nodes, and the edges between the nodes are labeled by an integer showing the angle between the mirrors (π/*n* radians or 180/*n* degrees). Circled nodes show which mirrors are active for each form; a mirror is active with respect to a vertex that does not lie on it.

Link to source web page

Row	Operation	Schläfli symbol	Symmetry	Coxeter diagram	Description
1	**Parent**	t_0{p,q,r}	[p,q,r]		Original regular form {p,q,r}
2	**Rectification**	t_1{p,q,r}			Truncation operation applied until the original edges are degenerated into points.
3	**Birectification (Rectified dual)**	t_2{p,q,r}			Face are fully truncated to points. Same as rectified dual.
4	**Trirectification (dual)**	t_3{p,q,r}			Cells are truncated to points. Regular dual {r,q,p}
5	**Truncation**	$t_{0,1}${p,q,r}			Each vertex is cut off so that the middle of each original edge remains. Where the vertex was, there appears a new cell, the parent's vertex figure. Each original cell is likewise truncated.
6	**Cantellation**	$t_{0,2}${p,q,r}			A truncation applied to edges and vertices and defines a progression between the regular and dual rectified form.
7	**Runcination (or expansion)**	$t_{0,3}${p,q,r}			A truncation applied to the cells, faces and edges; defines a progression between a regular form and the dual.
8	**Bitruncation**	$t_{1,2}${p,q,r}			A truncation between a rectified form and the dual rectified form.
9	**Bicantellation**	$t_{1,3}${p,q,r}			Cantellated dual {r,q,p}.
10	**Tritruncation**	$t_{2,3}${p,q,r}			Truncated dual {r,q,p}.
11	**Cantitruncation**	$t_{0,1,2}${p,q,r}			Both the *cantellation* and *truncation* operations applied together.
12	**Runcitruncation**	$t_{0,1,3}${p,q,r}			Both the *runcination* and *truncation* operations applied together.
13	**Runcicantellation**	$t_{0,2,3}${p,q,r}			Runcitruncated dual {r,q,p}.
14	**Bicantitruncation**	$t_{1,2,3}${p,q,r}			Cantitruncated dual {r,q,p}.
15	**Omnitruncation (runcicantitruncation)**	$t_{0,1,2,3}${p,q,r}			Application of all three operators.
16	**Snub**	s{p,2q,r}			Alternated truncation

1= 4	1	{1, 0, 0, 0}	Parent
2= 3	2	{0, 1, 0, 0}	Rectified
3= 2	3	{0, 0, 1, 0}	BiRectified
4= 1	4	{0, 0, 0, 1}	TriRectified
5= 10	5	{1, 1, 0, 0}	Truncated
6= 9	6	{1, 0, 1, 0}	Cantellated
7*	7	{1, 0, 0, 1}	Runcinated
8*	8	{0, 1, 1, 0}	BiTruncated
9= 6	9	{0, 1, 0, 1}	BiCantellated
10= 5	10	{0, 0, 1, 1}	TriTruncated
11=14	11	{1, 1, 1, 0}	CantiTruncated
12=13	12	{1, 1, 0, 1}	RunciTruncated
13=12	13	{1, 0, 1, 1}	CantiRuncinated
14=11	14	{0, 1, 1, 1}	BiCantiTruncated
15*	15	{1, 1, 1, 1}	OmniTruncated
16*	16	{0, 0, 0, 0}	Snub

0/1

x+y=15

The 24-cell family

Link to source web page

The 24-cell family consists of 16 members, 3 of which overlap with the tesseract/16-cell family because of the coincidence of the 24-cell with the rectified 16-cell. Furthermore, due to the fact that the 24-cell is self-dual, only 9 of the members of this family are distinct. The self-duality also causes some members (marked with *) of this family to have a higher degree of symmetry than the 24-cell itself.

One special member of this family, the snub 24-cell, has a diminished 24-cell symmetry. It is not derived by the usual uniform truncation processes, but by a particular partitioning of the 24-cell's edges in the Golden Ratio.

As BC4 (D4') only, but not "identical" with 24-cell dual (D4) in F4

1= 4 1. *(Dual 24-cell: identical to the [illegible])*
2= 3 2. *(Rectified dual 24-cell: identical to the rectified [illegible])*
5=10 3. *(Truncated dual 24-cell: identical to the truncated 24-cell.)*
3= 2 4. Rectified 24-cell: a pretty uniform polychoron bounded by 24 cubes and 24 cuboctahedra.
6= 9 5. *(Cantellated dual 24-cell: identical to the cantellated 24-cell.)*
8* 6. Bitruncated 24-cell*: a beautiful uniform polychoron bounded by 48 truncated cubes.
11=14 7. *(Cantitruncated dual 24-cell: identical to the cantitruncated 24-cell.)*
4= 1 8. 24-cell: the regular member of this family.
7* 9. Runcinated 24-cell*: a uniform polychoron bounded by 48 octahedra and 192 triangular prisms.
9= 6 10. Cantellated 24-cell: a pretty uniform polychoron bounded by 24 rhombicuboctahedra, 24 cuboctahedra, and 96 triangular prisms.
12=13 11. *(Runcitruncated dual 24-cell: identical to the runcitruncated 24-cell.)*
10= 5 12. Truncated 24-cell: a uniform polychoron bounded by 24 cubes and 24 truncated octahedra.
13=12 13. Runcitruncated 24-cell: a beautiful uniform polychoron bounded by 24 truncated octahedra, 24 rhombicuboctahedra, 96 triangular prisms, and 96 hexagonal prisms.
14=11 14. Cantitruncated 24-cell: a pretty polychoron bounded by 24 great rhombicuboctahedra, 24 truncated cubes, and 96 triangular prisms.
15* 15. Omnitruncated 24-cell*: a beautiful uniform polychoron bounded by 48 great rhombicuboctahedra and 192 hexagonal prisms.
16* 16. Snub 24-cell: a semi-regular polychoron bounded by 24 icosahedra and 120

All 16 $F_4=D_4$(24-cell dual) Coordinates with D_4'= BC_4(24-cell)

Vertex coordinates for all 15 forms are given below, including dual configurations from the two regular 24-cells. (The dual configurations are named in bold.) Active rings in the first and second nodes generate points in the first column. Active rings in the third and fourth nodes generate the points in the second column. The sum of each of these points are then permutated by coordinate positions, and sign combinations. This generates all vertex coordinates. Edge lengths are 2.

Link to source web page

#	Base point(s) t(0,1)	Base point(s) t(2,3)	Schläfli symbol	Name	Coxeter diagram
1 #1		(0,0,1,1)√2	{3,4,3}	24-cell dual	
2 #2		(0,1,1,2)√2	r{3,4,3}	rectified 24-cell dual	
3 #5		(0,1,2,3)√2	t{3,4,3}	truncated 24-cell dual	
10 #16/0		(0,1,φ,φ+1)√2	s{3,4,3}	snub 24-cell	
2 #3	(0,2,2,2) (1,1,1,3)		r{3,4,3}	Bi rectified 24-cell	
4 #9	(0,2,2,2) + (1,1,1,3) +	(0,0,1,1)√2 "	rr{3,4,3}	Bicantellated 24-cell	
8 #8	(0,2,2,2) + (1,1,1,3) +	(0,1,1,2)√2 "	2t{3,4,3}	bitruncated 24-cell	
5 #11	(0,2,2,2) + (1,1,1,3) +	(0,1,2,3)√2 "	tr{3,4,3}	cantitruncated 24-cell dual	

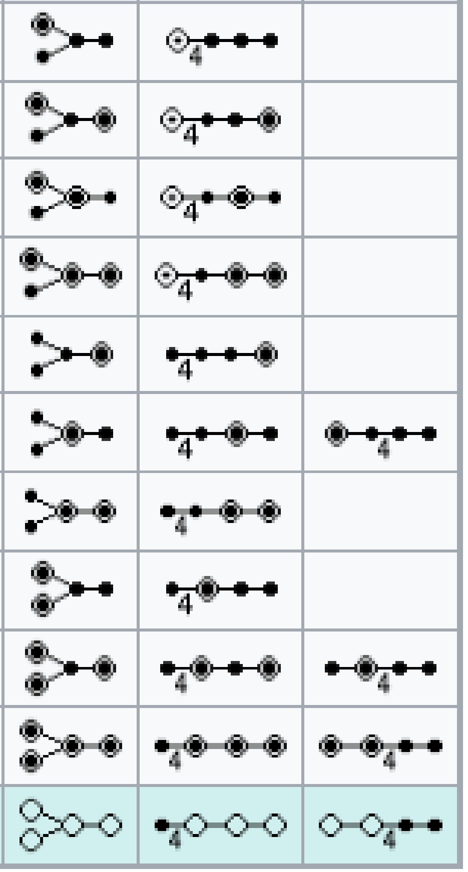
#	Base point(s) t(0,1)	Base point(s) t(2,3)	Schläfli symbol	Name	Coxeter diagram
1 #4	(0,0,0,2) (1,1,1,1)		{3,4,3}	TriRectified 24-cell	
7 #7	(0,0,0,2) + (1,1,1,1) +	(0,0,1,1)√2 "	$t_{0,3}${3,4,3}	runcinated 24-cell	
4 #6	(0,0,0,2) + (1,1,1,1) +	(0,1,1,2)√2 "	$t_{1,3}${3,4,3}	cantellated 24-cell dual	
6 #12	(0,0,0,2) + (1,1,1,1) +	(0,1,2,3)√2 "	$t_{0,1,3}${3,4,3}	runcitruncated 24-cell dual	
3 #10	(1,1,1,5) (1,3,3,3) (2,2,2,4)		t{3,4,3}	Tri truncated 24-cell	
6 #13	(1,1,1,5) + (1,3,3,3) + (2,2,2,4) +	(0,0,1,1)√2 " "	$t_{0,2,3}${3,4,3}	CantiRuncinated runcitruncated 24-cell	
5 #14	(1,1,1,5) + (1,3,3,3) + (2,2,2,4) +	(0,1,1,2)√2 " "	tr{3,4,3}	Bi cantitruncated 24-cell	
9 #15	(1,1,1,5) + (1,3,3,3) + (2,2,2,4) +	(0,1,2,3)√2 " "	$t_{0,1,2,3}${3,4,3}	Omnitruncated 24-cell	

11 D_4=BC_4 (16-cell parent) Coordinates

The *base point* can generate the coordinates of the polytope by taking all coordinate permutations and sign combinations. The edges' length will be √2. Some polytopes have two possible generator points. Points are prefixed by *Even* to imply only an even count of sign permutations should be included.

Link to source web page

BC4 (16-cell) Wythoffian #:	#	Name(s)	Base point	Johnson	Coxeter diagrams D_4	B_4	F_4
#4	1	$h\gamma_4$	Even (1,1,1,1)	demitesseract TriRectic			
#7	3	$h_3\gamma_4$	Even (1,1,1,3)	runcic tesseract			
#8	2	$h_2\gamma_4$	Even (1,1,3,3)	cantic tesseract BiTruncic			
#12	4	$h_{2,3}\gamma_4$	Even (1,3,3,3)	runcicantic tesseract			
#1	1	$t_3\gamma_4 = \beta_4$ F4 and BC4	(0,0,0,2)	16-cell			
#2	5	$t_2\gamma_4 = t_1\beta_4$	(0,0,2,2)	rectified 16-cell			
#5	2	$t_{2,3}\gamma_4 = t_{0,1}\beta_4$	(0,0,2,4)	truncated 16-cell			
#9	6	$t_1\gamma_4 = t_2\beta_4$	(0,2,2,2)	Bicantellated 16-cell			
#6	9	$t_{1,3}\gamma_4 = t_{0,2}\beta_4$	(0,2,2,4)	cantellated 16-cell			
#11	7	$t_{1,2,3}\gamma = t_{0,1,2}\beta_4$	(0,2,4,6)	cantitruncated 16-cell			
#16	8	$s\{3^{1,1,1}\}$	(0,1,φ,φ+1)/√2	Snub 24-cell			

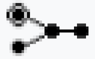
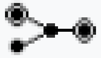
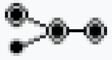
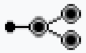
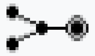
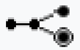
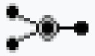
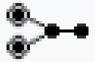
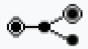
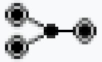
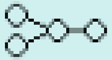


1	{1, 0, 0, 0}	Parent
2	{0, 1, 0, 0}	Rectified
3	{0, 0, 1, 0}	BiRectified
4	{0, 0, 0, 1}	TriRectified
5	{1, 1, 0, 0}	Truncated
6	{1, 0, 1, 0}	Cantellated
7	{1, 0, 0, 1}	Runcinated
8	{0, 1, 1, 0}	BiTruncated
9	{0, 1, 0, 1}	BiCantellated
10	{0, 0, 1, 1}	TriTruncated
11	{1, 1, 1, 0}	CantiTruncated
12	{1, 1, 0, 1}	RunciTruncated
13	{1, 0, 1, 1}	CantiRuncinated
14	{0, 1, 1, 1}	BiCantiTruncated
15	{1, 1, 1, 1}	OmniTruncated
16	{0, 0, 0, 0}	Snub

11 D_4=BC_4 (16-cell parent) Coordinates

The *base point* can generate the coordinates of the polytope by taking all coordinate permutations and sign combinations. The edges' length will be $\sqrt{2}$. Some polytopes have two possible generator points. Points are prefixed by *Even* to imply only an even count of sign permutations should be included.

Link to source web page

BC4	#	Name(s) (D4') Wythoffian #:	Base point	Johnson	Coxeter diagrams D4	D4 Dynkin #s (missing triality):
#4	1	$h\gamma_4$	Even (1,1,1,1)	demitesseract TriRectic		#4
#7	3	$h_3\gamma_4$	Even (1,1,1,3)	runcic tesseract		#8
#8	2	$h_2\gamma_4$	Even (1,1,3,3)	cantic tesseract BiTruncic		#7
#12	4	$h_{2,3}\gamma_4$	Even (1,3,3,3)	runcicantic tesseract		#12 (#11,#13) 11  13 
#2	1	$t_3\gamma_4 = \beta_4$ F4 and BC4	(0,0,0,2)	16-cell		#2 (#3) 3 
#1	5	$t_2\gamma_4 = t_1\beta_4$	(0,0,2,2)	rectified 16-cell		#1
#5	2	$t_{2,3}\gamma_4 = t_{0,1}\beta_4$	(0,0,2,4)	truncated 16-cell		#5 (#6) 6 
#9	6	$t_1\gamma_4 = t_2\beta_4$	(0,2,2,2)	Bicantellated 16-cell		#10 (#9) 9 
#6	9	$t_{1,3}\gamma_4 = t_{0,2}\beta_4$	(0,2,2,4)	cantellated 16-cell		#14
#11	7	$t_{1,2,3}\gamma = t_{0,1,2}\beta_4$	(0,2,4,6)	cantitruncated 16-cell		#15
#16	8	$s\{3^{1,1,1}\}$	$(0,1,\varphi,\varphi+1)/\sqrt{2}$	Snub 24-cell		#16

D_4 Uniform Polychora

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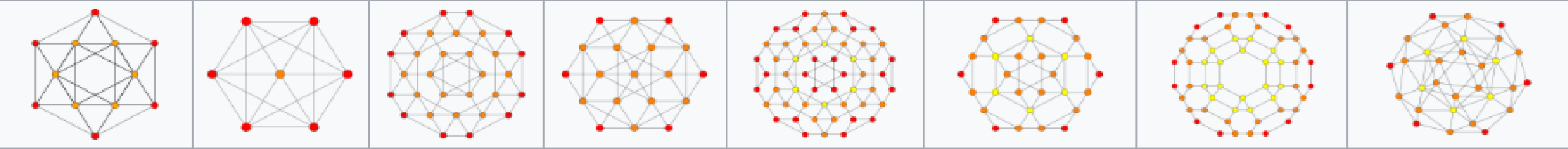
vertices: 24 96/24 288/192 288/192 576 576 1152 96

x+y=15

0/1

D4 uniform polychora (red index on Coxeter-Dynkin diagram indicates Wythoff operation table row detail) [hide]							
	triality:	triality:	triality:	triality:			
1	2 96	5 192	8 192	11 576	14	15	16
folding:	3 96	6 288	9 288	12 576	folding:	folding:	folding:
	4 24	7 144	10 192	13 576			
$r\{3,3^{1,1}\}$ $\{3^{1,1,1}\}=\{3,4,3\}$	$\{3,3^{1,1}\}$ $h\{4,3,3\}$	$t\{3,3^{1,1}\}$ $h_2\{4,3,3\}$	$2r\{3,3^{1,1}\}$ $h_3\{4,3,3\}$	$2t\{3,3^{1,1}\}$ $h_{2,3}\{4,3,3\}$	$rr\{3,3^{1,1}\}$ $r\{3^{1,1,1}\}=r\{3,4,3\}$	$tr\{3,3^{1,1}\}$ $t\{3^{1,1,1}\}=t\{3,4,3\}$	$sr\{3,3^{1,1}\}$ $s\{3^{1,1,1}\}=s\{3,4,3\}$



0/1

Palindromic Operations

1	{1, 0, 0, 0}	Parent
2	{0, 1, 0, 0}	Rectified
3	{0, 0, 1, 0}	BiRectified
4	{0, 0, 0, 1}	TriRectified
5	{1, 1, 0, 0}	Truncated
6	{1, 0, 1, 0}	Cantellated
7	{1, 0, 0, 1}	Runcinated
8	{0, 1, 1, 0}	BiTruncated
9	{0, 1, 0, 1}	BiCantellated
10	{0, 0, 1, 1}	TriTruncated
11	{1, 1, 1, 0}	CantiTruncated
12	{1, 1, 0, 1}	RunciTruncated
13	{1, 0, 1, 1}	CantiRuncinated
14	{0, 1, 1, 1}	BiCantiTruncated
15	{1, 1, 1, 1}	OmniTruncated
16	{0, 0, 0, 0}	Snub

D4 Uniform Polychora

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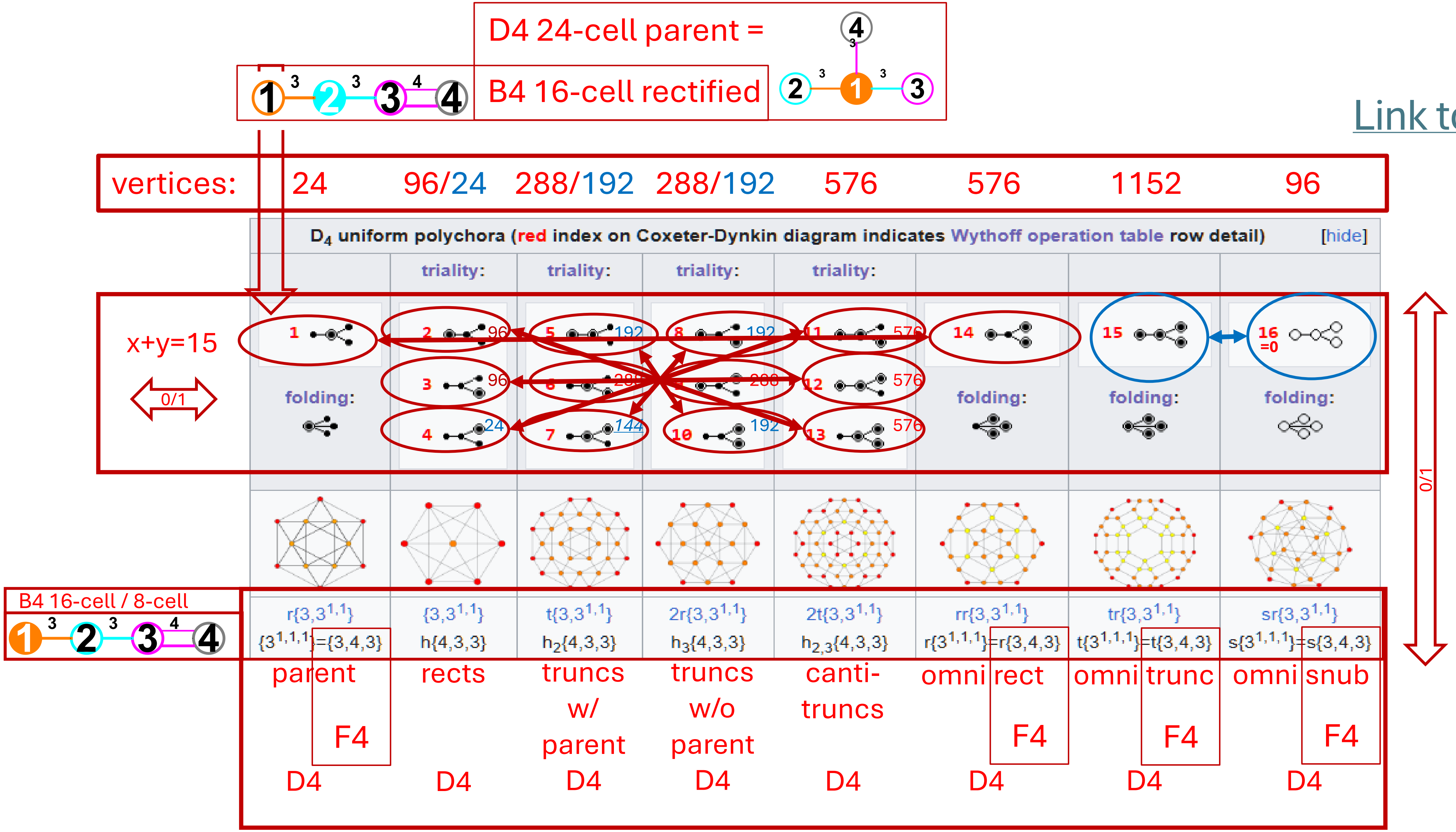

Palindromic Operations

1	{1, 0, 0, 0}	Parent
2	{0, 1, 0, 0}	Rectified
3	{0, 0, 1, 0}	BiRectified
4	{0, 0, 0, 1}	TriRectified
5	{1, 1, 0, 0}	Truncated
6	{1, 0, 1, 0}	Cantellated
7	{1, 0, 0, 1}	Runcinated
8	{0, 1, 1, 0}	BiTruncated
9	{0, 1, 0, 1}	BiCantellated
10	{0, 0, 1, 1}	TriTruncated
11	{1, 1, 1, 0}	CantiTruncated
12	{1, 1, 0, 1}	RunciTruncated
13	{1, 0, 1, 1}	CantiRuncinated
14	{0, 1, 1, 1}	BiCantiTruncated
15	{1, 1, 1, 1}	OmniTruncated
16	{0, 0, 0, 0}	Snub

1	{1, 0, 0, 0}	Parent
2	{0, 1, 0, 0}	Rectified
3	{0, 0, 1, 0}	BiRectified
4	{0, 0, 0, 1}	TriRectified
5	{1, 1, 0, 0}	Truncated
6	{1, 0, 1, 0}	Cantellated
7	{1, 0, 0, 1}	Runcinated
8	{0, 1, 1, 0}	BiTruncated
9	{0, 1, 0, 1}	BiCantellated
10	{0, 0, 1, 1}	TriTruncated
11	{1, 1, 1, 0}	CantiTruncated
12	{1, 1, 0, 1}	RunciTruncated
13	{1, 0, 1, 1}	CantiRuncinated
14	{0, 1, 1, 1}	BiCantiTruncated
15	{1, 1, 1, 1}	OmniTruncated
16	{0, 0, 0, 0}	Snub

The tesseract/16-cell family

Link to source web page

The tesseract/16-cell family consists of 15 members having the symmetry of the tesseract and the 16-cell. Due to the coincidence of the 24-cell with the rectified 16-cell, three of these 15 members coincide with members of the 24-cell family, thus leaving 12 unique members in this family.

1 1. 16-cell: one of the two regular members.
2 2. *(Rectified 16-cell: identical to the 24-cell.)*
5 3. Truncated 16-cell: a uniform polychoron bounded by 16 truncated tetrahedra and 8 octahedra.
3 4. Rectified tesseract: a uniform polychoron bounded by 16 tetrahedra and 8 cuboctahedra.
6 5. *(Cantellated 16-cell: identical to the rectified 24-cell.)*
8 6. Bitruncated tesseract (bitruncated 16-cell): a uniform polychoron bounded by 8 truncated octahedra and 16 truncated tetrahedra.
11 7. *(Cantitruncated 16-cell: identical to the truncated 24-cell; a uniform polychoron bounded by 24 truncated octahedra and 24 cubes.)*
4 8. Tesseract: the other regular member of this family.
7 9. Runcinated tesseract (runcinated 16-cell): a uniform polychoron bounded by 32 cubes, 32 triangular prisms, and 16 tetrahedra.
9 10. Cantellated tesseract: a uniform polychoron bounded by 8 rhombicuboctahedra, 16 octahedra, and 32 triangular prisms.
12 11. Runcitruncated 16-cell: a pretty uniform polychoron bounded by 8 rhombicuboctahedra, 16 truncated tetrahedra, 24 cubes, and 32 hexagonal prisms.
10 12. Truncated tesseract: a uniform polychoron bounded by 8 truncated cubes and 16 tetrahedra.
13 13. Runcitruncated tesseract: a pretty uniform polychoron bounded by 8 truncated cubes, 16 cuboctahedra, 24 octagonal prisms, and 32 triangular prisms.
14 14. Cantitruncated tesseract: a pretty polychoron bounded by 8 great rhombicuboctahedra, 16 truncated tetrahedra, and 32 triangular prisms.
15 15. Omnitruncated tesseract (omnitruncated 16-cell): a uniform polychoron bounded by 8 great rhombicuboctahedra, 16 truncated octahedra, 24 octagonal prisms, and 32 hexagonal prisms.

1	{1, 0, 0, 0}	Parent
2	{0, 1, 0, 0}	Rectified
3	{0, 0, 1, 0}	BiRectified
4	{0, 0, 0, 1}	TriRectified
5	{1, 1, 0, 0}	Truncated
6	{1, 0, 1, 0}	Cantellated
7	{1, 0, 0, 1}	Runcinated
8	{0, 1, 1, 0}	BiTruncated
9	{0, 1, 0, 1}	BiCantellated
10	{0, 0, 1, 1}	TriTruncated
11	{1, 1, 1, 0}	CantiTruncated
12	{1, 1, 0, 1}	RunciTruncated
13	{1, 0, 1, 1}	CantiRuncinated
14	{0, 1, 1, 1}	BiCantiTruncated
15	{1, 1, 1, 1}	OmniTruncated
16	{0, 0, 0, 0}	Snub

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1 1. 600-cell: one of the regular members of this family.

2 2. Rectified 600-cell: a beautiful quasiregular polychoron bounded by 120 regular icosahedra and 600 octahedra.

5 3. Truncated 600-cell: a beautiful uniform polychoron bounded by 120 regular icosahedra and 600 truncated tetrahedra.

3 4. Rectified 120-cell: a beautiful uniform polychoron bounded by 120 icosidodecahedra and 600 tetrahedra.

6 5. Cantellated 600-cell: a beautiful uniform polychoron bounded by 120 icosidodecahedra, 600 cuboctahedra, and 720 pentagonal prisms.

8 6. Bitruncated 120-cell (bitruncated 600-cell): a beautiful uniform polychoron bounded by 120 truncated icosahedra and 600 truncated tetrahedra.

11 7. Cantitruncated 600-cell: a beautiful uniform polychoron with 120 truncated icosahedra, 720 pentagonal prisms, and 600 truncated octahedra.

4 8. 120-cell: the other regular member of this family.

7 9. Runcinated 120-cell (runcinated 600-cell): a beautiful uniform polychoron bounded by 120 regular dodecahedra, 600 tetrahedra, 720 pentagonal prisms, and 1200 triangular prisms, for a whopping total of 2640 cells.

9 10. Cantellated 120-cell: a beautiful uniform polychoron with 120 rhombicosidodecahedra, 600 octahedra, and 1200 triangular prisms.

12 11. Runcitruncated 600-cell: a beautiful uniform polychoron bounded by 120 rhombicosidodecahedra, 600 truncated tetrahedra, 720 pentagonal prisms, and 1200 hexagonal prisms.

10 12. Truncated 120-cell: a cute uniform polychoron bounded by 120 truncated dodecahedra and 600 tetrahedra.

13 13. Runcitruncated 120-cell: a cute uniform polychoron with 120 truncated dodecahedra, 720 decagonal prisms, 1200 triangular prisms, and 600 cuboctahedra.

14 14. Cantitruncated 120-cell: a beautiful uniform polychoron with 120 great rhombicosidodecahedra, 1200 triangular prisms, and 600 truncated tetrahedra.

15 15. Omnitruncated 120-cell (omnitruncated 600-cell): the grand-daddy of them all, the largest convex uniform polychoron, having 120 great rhombicosidodecahedra, 720 decagonal prisms, 1200 hexagonal prisms, and 600 truncated octahedra.

16 16. Grand antiprism: an unusual polychoron bounded by 20 pentagonal antiprisms and 300 tetrahedra. It is the closest uniform 4D equivalent to the 3D family of antiprisms.